The Application of Interference Methods to Astronomy.

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THE recent measurement at the Mount Wilson Observatory, California, with the aid of an interferometer, of the angular diameter of the star Betelgeuse has attracted much attention, and has incidentally illustrated the advantages to be derived from the application of interference methods to astronomical measurement. In view of the striking success of this application, it is somewhat surprising that the possibilities of the method have been generally overlooked by astronomers, for the principles underlying the methods are by no means new, and their application to the determination of the angular diameters of the stars was indicated by Fizeau so long ago as 1868. It is of interest to recall the exact words used by Fizeau, the suggestion being thrown out by him incidentally in a report on the Bordin prize of the Académie des Sciences :—

> "Il existe, pour la plupart des phenomenes d'interférence, tels que les franges d'Young, celles des miroirs de Fresnel, et celles qui donnent lieu à la scintillation d'après Arago, une relation remarquable et nécessaire entre la dimension des franges et celles de la source lumineuse; en sorte que les franges, d'une ténuité extrême, ne peuvent prendre naissance que lorsque la source lumineuse n'a plus que des dimensions angulaires presque insensible; d'où, pour le dire en passant, il est peut-être d'espérer qu'en s'appuyant sur ce principe et en formant, par exemple, au moyen de deux larges fentes très écartées, des franges d'interférence au foyer des grands instruments destinés à observer les étoiles, il deviendra possible d'obtenir quelques données nouvelles sur les diamètres angulaires de ces astres."

Stephan was the first to attempt the determination of the angular diameters of stars in this way. He worked out an approximate theory, based upon elementary considerations, of the interference phenomena obtained in the focal plane of an objective when a uniformly illuminated circular disc, of small angular diameter α, is viewed through it, the objective being covered by an opaque screen in which are two parallel narrow rectangular apertures. The conclusion was arrived at that, in general, a series of parallel and equidistant interference fringes would be obtained, but that the fringes would disappear if the distance apart of the slits l satisfied the relationship $\alpha = \lambda/l$,[1] λ being the mean wave-length of the light. A determination of the distance apart of the slits for which the interference fringes disappeared is therefore sufficient to enable the angular diameter of the object to be deduced. The practical difficulty arises that in attempting to determine in this way the angular diameter of a star, the loss of light due to the restriction of the aperture to two narrow slits is so great that the fringes would in general be very faint. Stéphan removed this difficulty by showing that extended apertures could be used without serious error provided that they were equal and possessed two axes of symmetry at right angles to each other, one of these axes passing through the centres of the two apertures, and that their width was small compared with their distance apart.

With the 80-cm. Foucault refractor of the Marseilles Observatory, Stéphan, in 1874, examined Sirius and other stars. The fringes were obtained, but they did not vanish even with the maximum possible separation of the slits. The least diameter measurable by this method with this instrument was $0''{\cdot}16$, but from the appearance of the fringes Stéphan was able to conclude that "les expériences citées ne prouvent pas seulement que le diamètre apparent des étoiles examinées est inférieur à $0''{\cdot}16$, elles montrent encore que ce diamètre est une très faible fraction du nombre precedent."

The subject was taken up again by Michelson, who, in 1890, gave a more rigid theoretical discussion of the method than Stéphan had done. Three cases of interest were examined, and the principal results obtained may be summarised thus :—

(i) If the object is a circular disc of uniform brightness, of apparent angular diameter α, the series of interference fringes produced in the focal plane of the objective when the aperture is limited to two narrow rectangular and parallel slits will vanish when the distance apart of the slits l is given by $l = 1{\cdot}22\lambda/\alpha$.

(ii) If the object is not of uniform brightness this relationship is modified. The precise modification for any given law of variation of brightness can be easily determined. If, for instance, the illumination falls off towards the limb according to the law of darkening observed for the sun, the relationship becomes $l = 1{\cdot}33\lambda/\alpha$.

(iii) If the object is a double source, with an angular separation of the components of amount α, the fringes vanish for a distance apart of the slits given by $\frac{1}{2}\lambda/\alpha$, provided that the two components are of equal brightness, that their distance apart is large compared with their separate diameters, and that the length of the slits is perpendicular to the line joining the centres of the two sources.

The method has practical application in the measurement of the angular diameters of small bodies such as planetary satellites and asteroids, and more recently of the angular diameters of stars, and also in the measurement of the separations and position-angles of close double stars or spectroscopic binaries.

The angular diameters of small bodies such as satellites are usually measured with a filar micrometer. The measurement is possible only under conditions of the best atmospheric definition, and even then the probable error of observation is relatively large, since the width of the finest spider web is comparable with the linear dimensions of the image in the focal plane of the telescope. Using the interference method, it is found that the fringes can be well observed even under conditions of poor atmospheric definition, when the use of a filar micrometer would be impossible. The method has the further advantage that as the distance apart of the slits is varied, the separation which causes the fringes to vanish can be very precisely determined, so that the error of observation is greatly reduced. With small, faint objects, on the other hand, the loss of light arising from the use of narrow slits is serious. At the Paris Observatory an attempt was made to determine by this method the angular diameter of the major satellites of Jupiter, but the light was not sufficient to render the fringes visible. Hamy, therefore, extended the theory to the case in which the slits are of a width which is comparable with their distance apart. If the slits are rectangular, of width a and distance between their centres l, the formula obtained by Michelson for the distance corresponding to the vanishing of the fringes must be replaced by

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[1] A rigid mathematical investigation replaces this by the relationship $\alpha = 1{\cdot}22\,\lambda/l$.

$$l = 1{\cdot}22\,\lambda/a\{1 + 0{\cdot}765\,(a/l)^2\}.$$

Michelson and Hamy used the method for the measurement of the angular diameters of the major satellites of Jupiter. Michelson, in 1891, observing with the 12-in. equatorial at the Lick Observatory, used adjustable narrow slits. Hamy, in 1899, used the large equatorial *coudé* of the Paris Observatory, and prepared a series of screens of such dimensions that their width was one-third of their distance apart ($a = \frac{1}{3}l$), the widths being calculated so that the angular diameters, α, deduced from the above formula decreased by 0″·1 with successive screens. The screens for which the fringes became least distinct were found, and by interpolation the angular diameters of the satellites were estimated to 0″·01. The angular diameters so obtained, reduced to a distance of five units for Jupiter, were as follows:—

	I.	II.	III.	IV.
Hamy	0″·98	0″·87	1″·28	1″·31
Michelson ...	1·02	0·94	1·37	1·31

The agreement between the two series is very much better than would be obtained with micrometer observations.

The method does not appear to have been further employed until the past year, when, at Michelson's suggestion, it was tried with the 100-in. Hooker telescope at Mount Wilson. In view of the advantages of the method, this seems somewhat surprising; possibly it is due to an exaggerated idea of the difficulty of the observation. Besides the application to satellites and asteroids, the method might be employed for the measurement of the oblateness of such bodies as Mercury, which have no satellites from a study of the motion of which the oblateness might be theoretically deduced, and for which micrometrical observations are not sufficiently accurate. It can easily be shown that by rotating the slits into different orientations the corresponding angular diameters are determined.

At Mount Wilson the method has been applied to the measurement of the angular diameters of stars. Theoretical considerations have indicated that the stars of largest angular diameter are to be sought amongst the giant red, or M-type, stars, such as Betelgeuse, Arcturus, etc., but that for no star is the diameter likely to exceed 0″·05, a quantity scarcely within reach even of the 100-in. reflector at Mount Wilson. Michelson, in 1890, had, however, indicated the possibility of employing the method in conjunction with an interferometer, thereby enabling the original separation of the two beams to be increased very considerably. The arrangement used at Mount Wilson is shown in Fig. 1, and diagrammatically in Fig. 2. A steel girder, LL, 20 ft. in length is fixed across the upper end of the tube of the 100-in. telescope. Two adjustable plane mirrors, AA, reflect the light from a star along the girder to two other mirrors, BB, 4 ft. apart, which in turn reflect the light down the telescope tube to the mirror, the two pencils finally uniting, as shown in Fig. 3, and producing interference fringes in the focus of the eyepiece. To obtain the equality in the path of the two beams (which, for interference in white light to be observed, must be of an accuracy of 1/10,000 in.), an adjustable double wedge of glass is placed in the path of one of

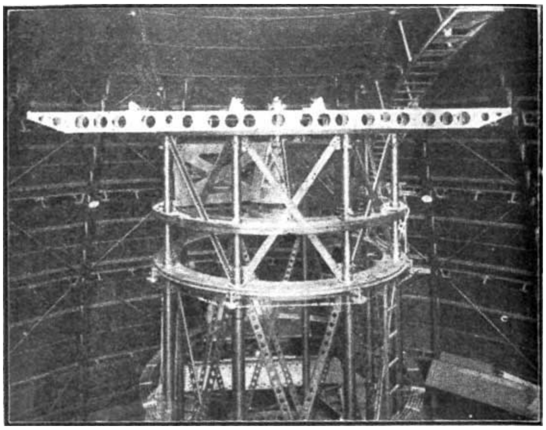

FIG. 1.—Stellar interferometer attached to end of tube of 100-in. telescope at the Mount Wilson Observatory.

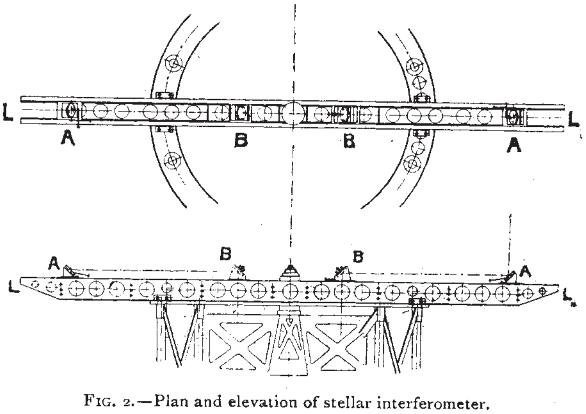

FIG. 2.—Plan and elevation of stellar interferometer.

the beams, compensated by a plane parallel plate in the other beam. The girder is capable of rotation, so as to test whether the fringes vanish in all position-angles, thus excluding the possibility of the vanishing being due to a double source. The observation calls for a high degree of experimental skill, as all who have used an interferometer will realise, and after shifting the mirrors it is a matter of considerable difficulty to find the fringes again.

With a base line of 20 ft. it should be possible to observe the disappearance of the fringes in the case of stars the angular diameters of which exceed about 0″·02. When the telescope was pointed on Vega the fringes did not disappear even when the two adjustable mirrors were at their maximum separation, indicating that the angular diameter of Vega is less than this amount. In the case of Betelgeuse the fringes disappeared when the separation of the mirrors was 10 ft. Adopting as the mean wave-length of the light 5500 angstroms, the value of λ/l is 0″·037. Assuming that there is no darkening towards the limb, the angular diameter of Betelgeuse will be 1·22 × 0″·037 or 0″·046. It is probable that the supposition of darkening according to the same law as for the sun would be nearer to the truth, and the angular diameter would then be 1·33 × 0″·037 or 0″·049. To this extent the deduced angular diameter is uncertain. More recently the fringes were found to vanish in the case of Arcturus when the mirrors were 19 ft. apart, corresponding to an angular diameter of 0″·024. This is almost the limiting angular diameter which can be measured with the present apparatus.

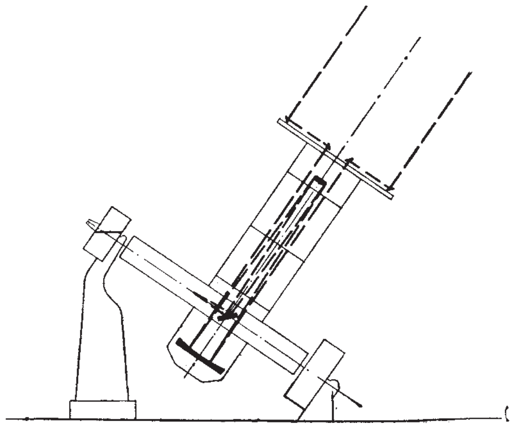

FIG. 3.—Diagram illustrating the paths of the two interfering beams.

The number of stars the angular diameters of which exceed 0″·02 is probably not very great; according to Eddington's estimate they are to be found amongst K-type stars of zero visual magnitude, or M-type stars of zero to third magnitude. The appearance of the fringes in the case of Vega with the maximum separation of the mirrors was so distinct, however, that it is considered that interference would be obtained with a base line of 100 ft. or more; if the mechanical difficulties can be overcome, the application of the method will no doubt be extended to base lines greater than 20 ft.

The determination of the angular diameters of stars is possible only with the aid of a very large instrument. For the measurement of the separations and position-angles of close double stars, the method can be employed with comparatively small instruments, and it is in this field that the method possesses the greatest possibilities. Its advantages as compared with the use of a filar micrometer are considerable. Adopting the late Lord Rayleigh's criterion for resolving power, a double star will appear just separated in a telescope if the central image of one component falls on the first diffraction ring of the other; since with a telescope of aperture d the distance between the central image and first diffraction ring is $1{\cdot}22\lambda/d$, it follows that this is the smallest angular separation of the components of a double star for which the star will appear double. But with the interference method, as has already been stated, the fringes disappear provided that the distance apart of the slits is $\lambda/2\alpha$, α being the angular separation of the components, the fringes due to one star then falling exactly between those due to the other. The least separation which can be observed by this method, corresponding to a distance apart of the slits equal to the aperture, d, is therefore $\lambda/2d$, and it will be seen that the method increases the resolving power of a telescope in the ratio of about 2·44 to 1. Expressing d in inches, the normal angular limit of resolution is approximately $5''/d$; with the 100-in. telescope a separation of 0″·05 should be just observable with normal methods of observation, the interference method reducing this to 0″·02. That this increase in resolving power is actually obtained is proved by the observations of Capella at Mount Wilson. This star was known to be a spectroscopic binary, but visual methods had failed to separate the components, though it was established that their separation could not exceed 0″·06; it therefore provided an interesting test object. The disappearance of the fringes was easily observed with the 100-in. telescope; the distance apart of the slits when this occurs determines the separation, and their orientation the position angle of the binary. Observations secured on six nights enabled an orbit to be computed. The observed distances and position-angles, together with the residuals from the values deduced from the computed orbit, are as follows:—

Date	Distance	Residual	Position-angle	Residual
	″	″	°	°
1919 Dec. 30	0·0418	0·00000	—	—
1920 Feb. 13	0·0458	−0·00003	5·0	+0·4
,, 14	0·0451	+0·00004	1·0	0·0
,, 15	0·0443	0·00000	356·4	−0·9
Mar. 15	0·0505	0·00000	242·0	−0·4
April 23	—	—	(not stated)	−0·2

These figures emphasise the remarkable accuracy of the method (the largest residual in distance is only four ten-thousandths of a second of arc!). It is stated that with care both angular separation and position-angle can be measured with an accuracy of about 1 per cent. This accuracy is much greater than is possible with a filar micrometer. The method possesses the further advantage that the percentage accuracy in both distance and position-angle does not decrease with decreasing separation; with the filar micrometer, on the other hand, the error of observation increases considerably with decreasing separation. For the successful use of the interference method good "seeing" is not essential, whereas for the observation of close double stars with a filar micrometer very good seeing is necessary. In explanation of this unexpected result it is suggested by Hale that "in bad seeing, when using the whole aperture of the objective, there is an integrated effect of the light-waves meeting in all possible phases, which tends to obliterate the details of the diffraction pattern of the star-image, but that when two light pencils are selected at opposite ends of a diameter the result is not an integration, but a mere displacement of the diffraction-pattern, sufficiently small for the eye to follow."

If the distance between the slits is greater than the value $l=\frac{1}{2}\lambda/\alpha$, which gives a unique position for which the fringes disappear, there will be four position-angles for the slits in which this occurs, these positions being symmetrical with reference to the line joining the two components, viz. $p\pm\theta$ and $p+\pi\pm\theta$, where p is the required position-angle. The most accurate method of observation is to adjust the distance apart of the slits so that θ is about 30° to 50°, and to measure the four positions in which the fringes vanish, so determining θ and p; if the separation of the slits is l, the value to use for the computation of the separation of the components of the binary is $l\cos\theta$, the separation therefore being $\lambda/2l\cos\theta$. If three different values of l are used and the corresponding values of θ are determined, the accuracy of the observation is increased. We have heretofore supposed that the components are equal in brightness and separated by a distance large compared with their diameters. If they are of unequal brightness the fringes do not completely disappear in any orientation of the slits, but instead the positions of minimum visibility (when the fringes of one star fall between those of the other) are observed. The variation in contrast is greater the more nearly equal the components are in brightness. The method is therefore suitable for the observation of close doubles which do not differ too greatly in brightness and are beyond the limit of resolution of, or observable only with difficulty and under the most favourable conditions with the telescope available, in conjunction with a filar micrometer. The time required for a single observation is longer, but this is more than compensated by the great increase in accuracy of the observation, by the possibility of observing under poor atmospheric conditions, and by the smaller number of observations required for the determination of an orbit. Prof. Hale expresses the hope that through a co-operative plan of observation, in which several observatories will take part, a large number of close binaries may be measured in this way.

An attempt is to be made at Mount Wilson to extend the method to the measurement of stars several minutes of arc apart. Until this is tried it cannot be asserted whether or not the difference of atmospheric disturbances along the optical paths of the two stars would prevent the fringes from being observed. If it proves feasible to observe them in this case it may become possible to measure the displacement of a star by the gravitational field of Jupiter, and thereby provide a further test of Einstein's theory, scarcely possible in any other manner. The method might then also be employed for the determination of stellar parallaxes and proper motions (which depend upon the differential displacements of adjacent stars) with a smaller probable error and in a shorter time than by existing methods. The further investigation of these possibilities of the method will be awaited with great interest.

The Paris Conference of the Museums Association.

FOR an association which, during the first thirty years of its existence, has confined its meetings to cities in the British Isles, the proposal to hold this year's conference in Paris seemed somewhat hazardous. Whatever objections may have presented themselves to some members, there can be no doubt that the experiment proved a greater success than any anticipated. During the week July 11–17 the seventy delegates from national, municipal, and semi-private museums, with their president, who, by good fortune, happened to be a man of such distinction as Sir Frederic Kenyon, were received in the most cordial manner by the heads of the State Museums of Art and of Science, by the Conseil Municipal and by the directors of its museums, and by the authorities of Les Invalides, the Bibliothèque Nationale, and similar institutions. Receptions at the Louvre, the Musée d'Histoire Naturelle, the Hôtel de Ville, and the club "Autour du Monde" enabled members to become personally acquainted with many French colleagues; and visits to the numerous and rich collections of Paris, Versailles, St. Germain, and Malmaison, under the guidance of distinguished authorities, with privileges accorded only to heads